Climate Policies in the Housing Market

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Abstract

Mitigating CO2 emissions in housing through retrofits has emerged as a crucial political issue. In this paper, we assess the macroeconomic and distributional impacts of key climate policies in the residential housing market. We build a quantitative heterogeneous agent model featuring high (green) and low (brown) energy efficient houses. Brown houses are associated with an additional cost of energy and can be retrofitted to a green house. We compare the effects of three policies: a tax on energy, a tax on brown rental income and a retrofit subsidy. The taxes widen the green to brown price ratio by penalizing brown houses, whereas the subsidy reduces it by lowering the substitution cost. The energy tax raises the user cost of brown housing, tightening affordability and increasing the renter share. The tax on brown rental income generates a "brown reallocation": by decreasing brown house prices while leaving the user cost unchanged, it induces low-income renters to transition into brown homeownership. Finally, the subsidy improves affordability, enabling low-income households to enter green homeownership.

JEL codes: E20, E60, H23, Q58, R31

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1 Introduction

Improving the energy efficiency of the housing stock is a priority for both climate mitigation and social equity. Residential housing is the second most carbon-intensive sector in the EU, accounting for 18% of total emissions. Simultaneously, energy affordability remains a critical issue, with one in ten European households struggling to adequately heat their homes. In response, the EU has set the objective of retrofitting[1] 15% of the worst-performing buildings by 2030. Achieving this target requires a rapid transformation of the housing stock, generating significant macroeconomic adjustments and distributional consequences.

Which policies can effectively achieve these targets? Existing policies have shown limited effectiveness, as retrofitting rates remain well below policy objectives.[2] Several factors explain this shortfall: many households face binding financial constraints and cannot afford the upfront cost of retrofitting; split incentives between landlords and tenants dampen renovation incentives; and the private profitability of retrofits is often too low.[3][4]

In addition, climate policies entail adverse distributional effects. Retrofit subsidies can disproportionately benefit wealthier households. Conversely, carbon taxes tend to be regressive, as the energy share of housing expenditure declines with income (Berry (2019)). Finally, policies targeting the rental market can also have adverse indirect effects on the renters. Quantifying these distributional impacts is therefore essential for evaluating and designing effective policy interventions.

In this paper, we develop a quantitative dynamic model of the low (brown) and high (green) energy efficient housing markets, in order to assess the macroeconomic and distributional implications of climate policies. We incorporate three key frictions related to retrofitting: high upfront cost of retrofitting, occasionally binding borrowing constraints and an institutional setting where renters cannot decide to retrofit. Households are heterogeneous in income, wealth and housing tenure (renter, owner occupier, landlord) and are subject to a borrowing constraint. Brown houses are associated with an additional cost on energy and can be retrofitted to green houses at a fixed upfront cost. We compare three climate policies: a tax

[1] We define retrofitting as renovation or insulation work (e.g. roof, wall, window) allowing to decrease the energy consumption of a dwelling.

[2] France and Germany allocate annually around 2 € billion in renovation subsidies, yet only achieve half of their targets. In France, despite 2.3 billion of subsidies and an objective of 200 thousand annual in-depth retrofits, only 91 thousand have been achieved in 2023. In Germany, between 2006 and 2020, 400 thousand annual renovations have been triggered for an annual cost of 2 billion euros, but the realization of energy savings is heavily debated. In Italy, the Superbonus 110% policy triggered around 500 thousand renovations in four years at a cost of around 215 billion euros.

[3] Astier et al. (2024) show that only 5% of the retrofits are actually profitable.

[4] We abstract from two other important frictions: information gap and supply constraint, described in Giraudet (2018).

on energy, a tax on brown rental income and a retrofit subsidy. They all relate with existing policies implemented in some European housing markets.[5] This framework allows to jointly study how the policies affect the access to home-ownership (renter/owner) and the energy efficiency of the market (green/brown).

The model extends the macro-housing model developed in Sommer and Sullivan (2018) by segmenting the housing market between energy efficiencies, and adding retrofitting. Houses are discrete goods that can be either green or brown and depreciate. Households derive utility from the consumption of a non-durable good and housing services. They face persistent idiosyncratic shocks to their inelastic supplied labor income. They make decisions about their non-durable consumption, housing services consumption, liquid savings, number of houses owned and their housing energy efficiency (brown or green). Houses are subject to transaction costs, and new housing construction is determined by a calibrated supply curve. Households endogenously decide to be renters, owner occupiers or landlords of a green or brown house. The model generates endogenous green and brown house prices and rents.

We examine the equilibrium implications of three distinct climate policies. First, we introduce a tax on energy (a carbon tax). Modeled as an increase in the relative price of energy, this policy raises the user cost of brown housing services. Second, we analyze a tax on brown rental income. This targeted instrument alters the return on brown housing for landlords — who represent 5% of households in our baseline calibration — thereby incentivizing retrofitting while aiming to insulate rental dwelling from the cost pass-through to rents. Third, we study a subsidy which reduces the fixed cost of retrofitting.

We calibrate the model to the French housing market in 2021, and match its key salient features: the green housing share (65%)[6], the renter share (41%) and the landlord share (14%). We conduct a steady-state analysis of three policies each designed to reach a green share of 75%. The policies are budget neutral following a revenue recycling scheme through an adjustment of the proportional income tax. Our findings reveal that these policies lead to highly heterogeneous effects in terms of prices and housing allocation.

The three policies achieve the environmental target through distinct equilibrium channels: both taxes increase the relative demand for green housing, while the subsidy increases its supply. This leads to opposing effects on the green-to-brown price ratio (green premium). Taxation policies widen the green premium, making green housing relatively less accessible to low income households. On the other hand, subsidizing retrofits decreases the premium,

[5]In particular, France legislated a gradual ban on renting low-efficiency dwellings in 2021. We capture this regulatory constraint as a tax on brown rental income.

[6]French dwellings are ranked on the Energy Performance Classification (EPC) between A and G. We define low efficient (brown) house is defined as a house with an EPC rating between E and G.

improving green affordability.

In particular, the subsidy induces a quite homogeneous substitution from brown to green housing across income levels, even increasing overall ownership rate. On the contrary, the taxes allow an increase in green ownership for the high income who are all able to substitute, while the low and middle tend to also reallocate towards other housing tenures.

The energy tax directly raises the user cost of brown housing. Combined with the increase in the green house price, low-income households are priced out of ownership, leading to an expansion of the renter share. In contrast, the tax on brown rental income decreases the share of brown landlord, not entirely compensated by an increase in green landlord. Overall, the supply of rental units decreases, driving up equilibrium rents. At the same time, the user cost of owner-occupied brown housing unaffected but the brown housing prices has fallen. Therefore, we find that low-income households substitute away from expensive rentals into cheaper brown ownership. This mechanism can be designated as brown reallocation, where former low-income renters reallocate to brown ownership.

Overall, while the taxes achieve the targeted increase in the green housing share by making green housing more *desirable*, the subsidy does so by enhancing its *affordability* through retrofitting.

This paper builds on the frameworks developed by the macro-housing literature (Piazzesi and Schneider, 2016; Sommer and Sullivan, 2018; Kaplan et al., 2020) and expands the research on climate policy for housing (Charlier et al., 2011; Giraudet et al., 2021; Astier et al., 2024). We contribute by building a quantitative setting that allows studying the interplay between climate policies, housing market dynamics, and inequalities. In addition, we contribute to the fast-growing literature on macroeconomic and distributional impacts of climate policies (Fried et al., 2021; Känzig, 2023; Kuhn and Schlattmann, 2024) by modeling specifically the housing market environmental transition. The closest paper to ours is Schlattmann (2024), which focuses on the spatial redistribution of carbon taxes. We contribute by modeling a detailed housing sector that explicitly captures rental market dynamics, landlordship and emphasize the role of financial frictions.

We show that climate policies in the housing market have different mechanisms and adverse distributional impacts, which paves the way for further studies on the climate policy mix. This study makes pioneering contributions to the literature by creating a novel macro model that examines the influence of climate policies on housing market transition. Additionally, it extends the macro climate literature by focusing on housing market policies. With housing playing a significant part in both CO2 emissions and household spending, this research enables us to understand and assess the distributional implications of future climate policies.

The rest of the paper is organized as follows. Section 2 situates the paper within the related literature and outlines its contribution. Section 3 presents the macro heterogeneous agent model with brown and green housing. Section describes the calibration strategy and the model properties. Section 5 compares the macroeconomic and distributional effects of each climate policy. Section 6 reports the perspectives of the project. Section 7 concludes.

2 Related literature

Climate policy in the housing market has been extensively studied in the public finance or environmental economics literature. However, this study focuses on its macroeconomic and distributional implications. It relies on three strands of the literature.

First, the literature on retrofitting in Environmental Economics has explored the energy efficiency gap and the factors influencing people's decisions to retrofit their homes. Giraudet and Missemer (2023) provides a literature review on this topic, examining why some individuals choose to retrofit while others do not. Distributional concerns in climate policy within the housing market have also been investigated. Bruegge et al. (2018) analyze the distributional impact of building energy codes and standards in California. They find that while these policies may be less cost-effective than taxes, they can be more effective in reducing energy consumption and lowering home prices, thereby benefiting lower-income households. Cayla et al. (2011), Berry (2019) and Douenne (2020) have explored the distributional consequences of various climate policies, such as carbon taxes and norms, taking into account the heterogeneous effects and responses depending on household income. Furthermore, Allcott (2015) discusses the efficiency of climate policies and highlights the challenge of low substitution elasticity among households. The issue of fuel poverty and incentives to retrofit has been examined by Charlier and Kahouli (2019). They show that incentives between landlords and tenants may lead to underinvestment. Saussay (2018) has developed an empirical and theoretical framework considering habits to study energy consumption in transport, highlighting the low substitution capacity of low income households. Other studies, such as Cajias and Piazolo (2012) and Cajias et al. (2016), have also explored the impact of energy performance on house prices and rents, providing insights into the relationship between energy efficiency and property values.

Closely related to our analysis, Vivier and Giraudet (2022) presents a building stock model of French dwellings with endogenous retrofitting dynamics. They emphasize the importance of retrofitting obligations to achieve net-zero energy targets in the residential sector. Their findings suggest that the retrofitting obligation is the most effective measure, outperforming other existing programs (subsidies, white certificate obligation, zero-interest loans, energy taxes) in triggering retrofits in private rental housing. The model considers different categories of property owners, including landlords and owner-occupiers. However, the housing market price reaction is not incorporated in the model's dynamics. Finally, Charlier et al. (2011) model in a general equilibrium framework the household investment decision to renovate, incorporating uncertainty regarding energy savings.

These studies provide valuable insights into the multifaceted aspects of housing climate policy, including its distributional implications, efficiency, and impact on energy consumption and fuel poverty. By building on and complementing this existing literature, this study aims to provide a framework able to capture the mechanisms through which climate policies impact the housing market, therefore assessing their efficiency and more broadly understanding their macroeconomic and distributional implications, building on the macroeconomic housing literature.

This study relies on the macroeconomics of the housing market with heterogeneous agents, drawing insights from the work of Sommer and Sullivan (2018), who develop an endogenous housing market model inspired by Gervais (2002). Piazzesi and Schneider (2016) review the theoretical and empirical literature on the macroeconomics of housing. It also relates to the literature on durable goods. Parodi (2022) demonstrates the ambiguity of optimal taxation for durable goods, as they serve as saving devices with implications for distribution, but the prices and resale values of durable goods are exogenous. Chafwehé (2017) focuses on market dynamics of durable goods, including endogenous prices. Additionally, some studies introduce the concept of switching quality in durable goods. Gavazza and Lanteri (2021) present a heterogeneous agents framework with car holders and a probability of decrease in car quality, leading to car ownership decisions. This is relevant to our analysis as houses can exhibit different "colors" due to household choices, with the ability to switch from brown to green.

By integrating insights from these various sources, the aim of this study is to integrate the climate dimension through brown and green housing in order to explore the optimal climate policy in the housing market.

Finally, the recent literature has increasingly focused on exploring the macroeconomic and distributional implications of climate policies in general equilibrium, such as Känzig (2023), Barrage (2020), Pedroni et al. (2022), Kuhn and Schlattmann (2024) or Fried et al. (2021) among others. Focusing on the housing market, empirical studies have indicated that energy efficiency has a discernible impact on house prices, although the differential is relatively small. Notably, Ferentinos et al. (2021) and Schuetze (2020) have evaluated the impact of transition risk on the housing market. Ferentinos et al. (2021) specifically assess the effects of norms in England and other climate policies on house prices and the resulting distributional consequences, while Schuetze (2020) evaluates the risk in residential mortgages, comparing a "green" and a "brown" portfolio in Germany. Finally, the closest paper to ours is Schlattmann (2024), that focuses on the spatial redistribution of carbon taxes. We contribute by modeling both saving decisions and a detailed housing sector that explicitly captures rental market dynamics and landlordship. We show that available climate policies in the housing market have different mechanisms and adverse distributional impacts, which paves the way for further studies on the climate policy mix.

The contribution of this study to the literature lies in providing a theoretical framework for analyzing the macroeconomic implications of climate policy in the housing sector, particularly its heterogeneous effects on households. Although studies like Fried (2022), Bakkensen and Barrage (2017), and Barrage and Furst (2019) have focused on housing in the context of physical risks and climate damages, there has been a lack of macroeconomic analysis of the housing market concerning transition risk and climate policy. Our work seeks to fill this gap by focusing on the housing market dynamics and distributional consequences within the context of climate policy.

By examining the impacts of different climate policies—such as energy taxes, brown rental income taxes, and retrofit subsidies—on housing allocation, prices, and ownership patterns, we offer new insights into how these policies differentially affect renters, homeowners, and landlords, particularly among low-income households. We highlight the varied distributional consequences of climate policies within the housing market, emphasizing their potential to alter housing access and affordability across income groups.

3 Model

We develop a dynamic incomplete-markets model extending the framework of Sommer and Sullivan (2018) to incorporate heterogeneous housing energy efficiency. Time is discrete and infinite. We formulate the problem recursively and denote next-period variables with a prime. The economy is populated by households who value non-durable consumption c and housing services x, and who supply labor inelastically subject to uninsurable idiosyncratic productivity risk (Aiyagari, 1994). Housing is a discrete durable good indexed by efficiency type $\theta \in \{b, g\}$. Brown houses ($\theta = b$) entail an additional flow energy cost paid by the occupier, but they can be upgraded to green ($\theta = g$) via a retrofit technology subject to investment costs paid by the owner. Households endogenously choose their tenure status. In particular, homeowners may choose to supply housing services to the rental market, in which case they are landlords. In equilibrium, prices and rents adjust to clear the green and brown housing and rental markets, and the government balances its budget with a proportional income tax.

3.1 Housing environment

Housing is a discrete durable good available in two efficiency levels, $\theta \in \{b, g\}$ and can be bought at unit price q_θ. We denote the stock of housing owned by a household as h_θ, which takes values on a finite grid $\mathcal{G}_h$. The grid index represents the number of houses owned by a given household.

$$h_\theta \in \{0, h_{1,\theta}, ..., h_{K,\theta}\} \tag{1}$$

Houses are costly to buy and sell. They pay non-convex transaction costs on the value of housing bought, respectively $\tau_x q_\theta h'_\theta$ and $\tau_s q_\theta h_\theta$. It generates sizable inaction regions with respect to the household decision to buy or sell, implying that only a fraction of the housing stock is traded in any given period. Furthermore, housing is subject to stochastic depreciation. An owner of a housing stock of size $h_{\theta,k}$ is facing the probability Δ of owning a house stock of size $h_{\theta,k-1}$ in the next period.

Housing services A linear technology transforms one unit of owned housing stock, h'_θ, into one unit of housing services, x_θ. Households can choose not to own a house and to instead purchase housing services, x_θ, in the rental market. Then, they are designated as renters. Renters can rent any of the larger shelter sizes on the housing service grid $\mathcal{G}_x$. Households may rent a small unit of housing service, $\underline{x}_\theta$, that is smaller than the minimum house size that is available for purchase, so that $\underline{x}_\theta < h_{1,\theta}$. This assumption is common in the housing literature (Sommer and Sullivan, 2018; Kaplan et al., 2020) and allows all households to consume a minimum of housing service but favors more flexibility for renters.

$$x_\theta \in \{\underline{x}_\theta, h_{1,\theta}, ..., h_{K,\theta}\} \tag{2}$$

Housing status Tenure status is defined by the difference between the housing stock owned h', and services consumed x $(h' - x)$. Households with $h' = 0$ are renters. Households with $h' > 0$ are homeowners, who may either consume their entire stock ($x = h'$, owner-occupiers) or lease a portion to the rental market ($x < h'$, landlords). Landlords incur a fixed management cost ϕ per period. Therefore, the supply of rental units is endogenously determined by landlords.

Energy from brown housing Brown and green housing services are perfect substitutes in utility. The only distinction is an additional user cost associated with brown housing services, representing energy expenditures. We normalize the energy cost of green housing to zero, so $p_e e$ represents the efficiency cost differential, where p_e denotes the price of energy or energy premium. Energy consumption is defined as a strictly complementarity to the consumption of brown housing services. We assume a linear technology where energy usage is one-to-one with services consumed (Equation 3).

$$p_e e = p_e f(x_b) = p_e x_b \tag{3}$$

The burden of energy costs falls on the user of the housing services. Therefore, renters and owner-occupiers of brown housing face an additional cost $p_e x_b$. A landlord owning brown property pays energy costs only on the portion of the stock they occupy personally, while the tenant covers the energy costs associated with the rented portion. This cost differential affects both equilibrium house prices and rents.

Retrofitting technology Brown owners can use a retrofitting process to convert their brown houses into green ones, while maintaining a constant housing stock. Retrofitting decisions generate a key connection between brown and green housing stock without relying on market prices, which in turn influences equilibrium prices. Retrofitting brown housing incurs a fixed cost (Equation 4).

$$\Gamma(h_b) = \gamma h_b \tag{4}$$

Where γ is the fixed cost in terms of the numeraire consumption good. Retrofitting costs increase linearly with the number of housing units owned. It corresponds for instance to the renewal of the roof, changing windows, or isolating the walls. This assumption of a non-convex cost is determinant in the model, as it generates lumpy retrofitting behavior.

3.2 Households

Heterogeneous households can invest into two types of assets: liquid savings (a) and illiquid housing (h_θ). In particular, a household can buy either a green or a brown house (h_g or h_b). We assume that households can only own one type of housing in a current period.[7] Below,

[7]This restriction is motivated by tractability and the dimensionality of the model. Allowing mixed housing portfolios would increase dramatically the number of discrete choices. A key consequence of this assumption is that retrofitting is an all-or-nothing decision applying to the household's entire housing stock. We plan to relax

the index on the house h_θ is only displayed when needed. Households can be categorized in five housing status, described in Figure 1.

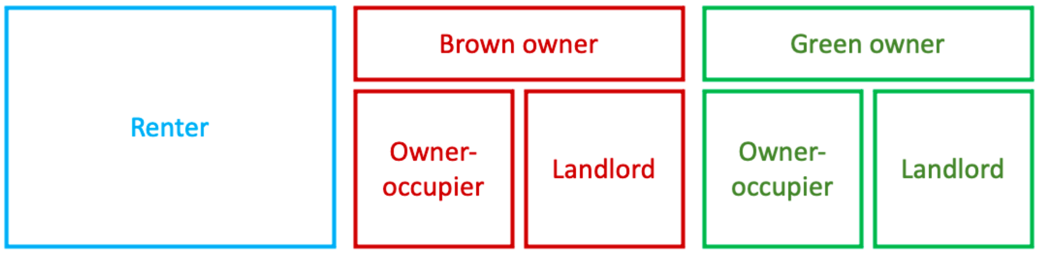

Figure 1: Housing status

Preferences Similar to Sommer and Sullivan (2018), households receive utility from non-durable consumption c and housing services x, the latter can be obtained either through renting or ownership.

$$U(c, x)$$

Households are risk averse, discount the future at rate β and maximize their intertemporal utility. As described previously, households are indifferent in terms of utility between brown and green housing. Their decisions in investing into brown or green comes from their intertemporal utility maximization.

Labor and capital income Each household is endowed with one unit of labor which is supplied inelastically. Household earn labor income ws, where w denotes the market wage and s is the household 's idiosyncratic labor productivity draw realized at the beginning of the period. We assume that labor productivity follows a log(AR1) process, and write the overall probability of moving from state s to state s' is denoted by $\pi^s(s'|s)$.

$$\log(s') = \varphi \log(s) + \varepsilon_s \qquad \varepsilon_s \sim \mathcal{N}(0, \sigma^2)$$

Households earn an interest rate r on their assets holdings a. Moreover, they face a borrowing constraint, where $\chi > 0$, implying that they can get indebted.

this assumption in future versions of this paper, and explore the potential consequences in the last section of the paper.

Household problem Households enter each time period with a stock of owned housing, $h \geq 0$ and accumulated assets $a' \geq -\chi$. Each household observes its idiosyncratic wage shock, s. Formally, households are described among the states of productivity s, the level of assets a, and housing h. Here, housing is decomposed into the house size h (with $h = 0$ if the household is a renter), and the house color θ. We consider that the house size actually repressents the number of houses owned by the household. As we will explain below, the renter does not choose in practice the color of housing. Let $\omega = (s, a, h, \theta)$ denote the vector of household state variables and household characteristics.

Given the current prices (q_b, q_g, ρ) and the household current states, each household chooses consumption c, next period assets a', next period house size before depreciation h', next period color θ' and housing services x, according to the following problem. The budget constraint displayed just below is exhaustive, implying that it incorporates all the possibilities for all states of households.

$$V(s, a, h, \theta) = \max_{c,x,a',h',\theta'} U(c, x) + \beta \sum_{s'} \pi^s(s', s) \sum_{\Delta} \pi^{\Delta}(\Delta) V(s', a', h', \theta') \tag{5}$$

subject to

$$\begin{aligned} sw + (1+r)a \quad &\geq \quad c + a' + \mathbb{1}_{\{x_b>0\}} p_e x_b \\ &+ \sum_{\theta \in \{b,g\}} \left[\rho_\theta (x_\theta - h'_\theta) + \mathbb{1}_{\{h'>x\}} \times \phi_\theta \right] \\ &+ \sum_{\theta \in \{b,g\}} \left[\mathbb{1}_{buy} \times q_\theta (1 + \tau_{buy}) h'_\theta - \mathbb{1}_{sell} \times (1 - \tau_{sell}) q_\theta h_\theta \right] \\ &+ \mathbb{1}_C \times \Gamma(h_b) \end{aligned}$$

and the additional feasibility constraints

$$\begin{gathered} h'_\theta \in \mathcal{G}_h, \quad x_\theta \in \mathcal{G}_x \quad \text{and} \quad x_\theta \leq h'_\theta \quad \text{if} \quad h'_\theta > 0 \\ a' \geq -\chi, \qquad c \geq 0 \end{gathered}$$

The left hand-side displays the labor income sw, which follows the process described above. Households earn interest income on their holdings of assets in the previous period $(1 + r)a$. Then, two first components of the right hand side are consumption and savings. As explained previously, the brown housing user pays an additional cost on energy $p_e e$, where $e = x_b$ if the house used is brown and $e = 0$ if the house is green. The terms $\rho_\theta(x_\theta - h_{j'})$

represents either the rental payment if the household is a renter (with $h'_\theta = 0$), or the rental income if the household is a landlord (then the term would turn negative). If the household is an owner-occupier, than this equals to zero. The term $q_\theta(h'_\theta - h_\theta)$ captures the difference between the value of the housing purchased at the start of the time period (h'_θ) and the stock of housing that the household entered the period with (h_θ). Transaction costs enter into the budget constraint if housing is sold ($\tau_s q_\theta h_\theta$) or bought ($\tau_x q_\theta h'_\theta$), with the binary indicators $\mathbb{1}_s$ and $\mathbb{1}_x$ indicating the events of selling and buying, respectively. The term ϕ_θ represents the fixed cost incurred by either brown or green landlord, with the binary indicators $\mathbb{1}_{h'>x}$, informing whether the household is landlord. Finally, if the household is a brown owner, he can convert his house into a green, paying the converting cost $\Gamma(h_b)$. Finally, the household is subject to a borrowing limit and the final equation expresses that an owner cannot consume more housing services than the one provided by his house.

Discrete Choices This problem incorporates several discrete choices: housing stock h' (Equation 1), housing services x (Equation 2), and housing energy efficiency θ. These decisions lead to five different housing status: renter, brown owner and green owner. Then, the owner can be either owner occupier or landlord, as described in Figure 1. Depending on her housing status, a household faces limited possibilities. A renter can choose to stay a renter, to buy a brown or a green house. A green owner has the choice between selling the house and being a renter or non adjusting. Finally, the brown owner can between sell and be a renter, non adjust or retrofit his current house into a green one. This means that in order for an owner to change a house, he would first need to sell it and become a renter for one period, and then to buy a new house in the next period.[8] Therefore, the housing status is crucial to understand how the households will be impacted by the policy.

Algorithm We compute the stationary policy functions by exploiting the recursive structure of the household problem. The combination of discrete housing choices and continuous consumption-savings choices introduces non-convexities in the value function, implying that first-order conditions are insufficient to identify the global optimum. At each iteration, we use an Endogenous Grid Method that combines the Euler Equation with an upper-envelope on the value function (Druedahl, 2021), to recover the optimal consumption-saving behavior. Then, we use extreme value taste shocks to maximize a smoothed expected value function on the discrete housing decisions (Iskhakov et al., 2017). The structure of the algorithm is further described in the Appendix.

[8] This assumption helps the computational tractability, as it reduces the number of discrete choices, but does not modify heavily the model's behavior.

3.3 Housing supply

We introduce a tractable housing supply sector. It is represented by an exogenous residential investment, I_θ, defined below

$$I_\theta = f(q_\theta, \epsilon)$$

where $f(q_\theta, \epsilon)$ is the constant elasticity supply function for residential investment, and the parameter ϵ represents the elasticity of residential investment with respect to the house price (q_θ). This suggests that brown and green housing share the same cost function. However, one could think that brown housing are cheaper to produce. Nonetheless, a similar supply curve limits as much as possible the distinction between brown and green, to only capture the difference in energy costs. As the cost curve is assumed to be convex, both will always be produced simultaneously.

Aggregate housing stock A linear technology translates residential investment into housing, so the law of motion for the aggregate stock of housing is a standard capital accumulation equation, including the retrofitting between brown and green housing.

$$H'_b = (1-\delta)\left[H_b - R + I_b\right]$$

$$H'_g = (1-\delta)\left[H_g + R + I_g\right]$$

H_θ is the current aggregate stock of housing, R represents the brown houses retrofitted to green houses. The aggregate stock of housing depreciates at rate δ, at the end of the period.[9] At steady state, the stock of housing is constant, such that the new investment compensates the depreciated stock of housing and the retrofitting.

$$I_b^* = \delta H_b^* + R^* \quad \text{and} \quad I_g^* = \delta H_g^* - R^*$$

3.4 Climate policies

A government sets the level of three climate policies: the carbon tax (τ_e), the percentage subsidy on retrofit (η), the tax on brown rental income (τ_b) introduce three climate policies in the

[9]The aggregate housing stock depreciation δ can be derived from the aggregation of idiosyncratic depreciation shocks to homeowners' housing stock Δ. We assume that depreciation shocks occur at the end of the period, after markets have cleared.

model. They modify the household problem described in Equation 5 in the following way.

- The carbon tax increases the energy premium. It affects the flow cost paid by all users of brown house services.

$$\mathbb{1}_{x_b>0}(p_e+\tau_e)x_b$$

- The tax on brown rental income reduces the income of brown landlords on their supply of housing services.

$$\mathbb{1}_{h'_b>x_b}(1-\tau_b)\rho_b \times (h'_b - x_b)$$

- The retrofit subsidy is a proportional decrease to the retrofit cost. It lowers the cost of upgrading a given brown housing stock for homeowners.

$$\mathbb{1}_C(1-\eta)\Gamma(h_b)$$

The government runs a balanced budget at each period, by adjusting a proportional income tax (τ_i) given a climate policy (Equation 6).

$$\int \tau_i(z+ra)d\lambda + \int \tau_e x_b(\omega)d\lambda + \int \tau_b\rho_b\left(h'_b(\omega)-x_b(\omega)\right)d\lambda_{h'>0} = \eta\int\Gamma\left(h'_g(\omega)\right)d\lambda_{h_b>0} \tag{6}$$

3.5 Market clearing

No-arbitrage condition on the rental market In equilibrium, we can infer a non arbitrage condition between brown and green rents. Indeed, the brown and green units of housing services provide the same utility, thus renters are indifferent between renting brown and green, as long as their expenditures are equal. A brown renter pays $\rho_b + p_e$ per unit of housing service as a rent (as we have $e = x_b$ from Equation 3, i.e. the energy is equal to the amount of housing services), whereas a green renter pays ρ_g per unit of housing services. Thus, this condition can be expressed as:

$$\rho_b + p_e = \rho_g \tag{7}$$

Consequently, only one rental price needs to be solved at equilibrium. Together with Equation 7, they determine the two rental market clearing.

Market clearing The baseline economy is at its stationary equilibrium where market prices are constant at their steady-state values (ρ, q_g, q_b). The per-capita housing stocks must remain constant. The no arbitrage condition on the rental market holds. λ denotes the distribution of individuals over the state space. Let $\omega = (s, a, h, \theta)$ denote the vector of household state variables and household characteristics.

This results into three market clearing conditions: brown and green housing markets (Equation 8) and the rental market (Equation 9).

$$\int h_b'(\omega) d\lambda = H_b' \quad \text{and} \quad \int h_g'(\omega) d\lambda = H_g' \tag{8}$$

$$\int (h_\theta'(\omega) - s_\theta(\omega)) d\lambda = 0 \tag{9}$$

More specifically the rental market clearing is defined such that the residual from the total stock of housing services minus housing services of both brown and green consumed by the homeowners are equal the housing services consumed by the renters. It can than also be written as:

$$\sum_\theta \int (h_\theta'(\omega) - x_\theta(\omega)) d\lambda_{h>0} = \int x(\omega) d\lambda_{h=0}$$

Where housing services supplied by landlords (left hand-side) equal the ones consumed by the renters (right hand-side).[10]

Incentives between brown and green housing In the model, owning a brown house compared to a green house comes with three disadvantages. First, there are recurring energy costs proportional to the size of the occupied house. Second, landlords of brown houses receive a lower rental income due to the non-arbitrage condition. This creates compelling incentives for brown homeowners to transition to green houses. Consequently, the prices of green houses are consistently higher than their brown counterparts, leading to reduced need for precautionary savings.

There are two possible ways to transition from brown to green ownership. The first option involves selling the brown house and purchasing a green one. However, this path incurs transaction costs, price differentials, and a transition to renting for at least one period. The second option entails converting the existing house to green, requiring payment of retrofitting costs.

[10] The no-arbitrage condition on the rental market (Equation 7) implies that equalizing the overall supply of housing services by landlords and its demand in the rental market is equivalent to clearing both rental markets.

3.6 Stationary equilibrium

A stationary recursive competitive equilibrium is defined as follows. Let $\omega = (s, a, h, \theta)$ denote the vector of household states and characteristics. Let $\Omega = (\tau_e, \tau_b, \eta)$ be the exogenous vector of climate policy instruments. Let λ be the cross-sectional distribution over the household states and characteristics. A stationary recursive competitive equilibrium is defined, for each type of household $\omega = (s, a, h, \theta)$ in a collection of value function $v(\omega)$ and policy functions for the households $\{c(\omega), a'(\omega), h'(\omega), \theta'(\omega), x(\omega)\}$, a price vector (ρ_*, q_{b*}, q_{g*}) and stationary distributions λ such that:

1. Given prices (ρ^*, q_b^*, q_g^*), and climate policies Ω, the policy functions $c(\omega), a'(\omega), h'(\omega), \theta'(\omega)$ and $x(\omega)$ are optimal decision rules to the households' decision problem presented in equation 5.

2. Rental, brown housing and green housing markets clear (Equations 8 and 9).

3. The government balances its budget using the proportional income tax τ_i (Equation 6).

4. The distribution of households λ is time invariant.

4 Calibration

We calibrate the model to the French housing market. The time period in the model is one year. The model is calibrated in two steps. In the first stage, a number of parameter values are drawn from other studies or obtained directly from data sources. In Table 1 the first stage parameters determined in this manner are designated as external calibration. Then, the rest of the parameters are calibrated by matching simulated moments from the model to empirical moments, mainly the homeownership rate. The empirical moments targeted during calibration are listed in Table 2.

4.1 Calibration

Data sources We calibrate the model to the French housing market in 2021. We extract most aggregate moments from the French Statistical Office (Insee) and the French Statistical Office for Energy (SDES). Moreover, in the following section, the French annual median wage is used to convert the monetary units of the model[11].

[11] The French annual median wage in 2019 is 22040 € according to INSEE (2021)

	Parameter	Value
External calibration		
Risk aversion	μ	2
Discount factor	β	0.92
Depreciation rate	δ	0.025
Supply elasticity of housing	ϵ	0.5
Transaction cost of buying	τ_x	0.05
Transaction cost of selling	τ_s	0.05
Autocorrelation of labor income shocks	φ	0.9
Standart deviation of labor income shocks	σ	0.2
Fixed cost of retrofitting	γ	0.45
Internal calibration		
Annual interest rate	r	0.03
Consumption share	α	0.727
Fixed cost of landlord	ϕ	0.1
Energy premium	p_e	0.0075
Housing supply productivity	ψ	0.009
Variance of taste preference (house size)	ν_{size}	0.008
Variance of taste preference (color)	ν_{color}	0.05
Variance of taste preference (housing status)	ν_{status}	0.06
Preference for green homeownership	μ_{color}	0.045

Table 1: Calibration

Household preferences and income Households value non durable consumption, c, and consumption of housing services, x, using a non-separable utility function.

$$U(c, x) = \frac{(c^{\alpha} x^{1-\alpha})^{\mu} - 1}{1 - \mu}$$

The risk aversion parameter μ is set to 2. The weight on non durable consumption α is 0.727. Finally, utility is discounted using the discount factor β, which value is set at 0.92, that is consistent with annual time-steps.

Besides, following many papers in the quantitative macro literature the stochastic process for household labor market productivity s is modeled with an AR(1) process. The autocorrelation of income shocks φ is set to 0.9 and the variance of σ is 0.2 (Sommer and Sullivan, 2018). The labor income process is discretized to seven states following Tauchen (1986). Finally, the annual risk free interest rate r is calibrated to 0.03, to match the liquid wealth over total income ratio.

Housing environment The functional form of the housing supply is defined as:

$$I(q_\theta) = \psi \times \left[q_\theta\right]^\epsilon$$

The supply elasticity of housing ϵ is calibrated to 0.5, following the estimates on French data from Chapelle et al. (2023). Therefore, housing has a low price-elasticity and supply slowly adapts to the demand. The parameter ψ is set at 0.009. According to Piazzesi and Schneider (2016) the yearly depreciation rate of housing is between 1.5 and 3 percents. We set the stochastic depreciation Δ to match an aggregate depreciation rate δ of 1.5 percent. Therefore, only 1.5 percent of the housing stock is renewed in each period. Following usual estimates on French housing data, the transaction cost of buying τ_x and selling τ_s are set equally to 0.05. In the baseline calibration, the fixed cost of both brown and green landlords is equal to 0.1 to match the share of landlords. Finally, the model incorporates three different housing stock size, designated as "one house", "multi-homeowner" and "large multi-homeowner". We set the grid unevenly to match the share of renters, homeowners possessing one house, homeowners possessing two to four dwellings, and large multi-homeowners (more than five dwellings).

Retrofit technology The calibration of the retrofit technology as described in equation 4 is particularly difficult, as the cost varies according to the house size, and many other characteristics (house or apartment, initial energy efficiency, among others). We set fixed retrofit cost per housing unit γ is set to 0.45, which corresponds to a monetary cost of 23k€ per housing unit.[12]

Energy cost differential The energy cost different, defined as p_e, plays a crucial role in the model as it is the main distinction between green and brown houses. Consequently, its calibration is a complex task, as it determines the varying expenses associated with living in either a brown or green house, exerting influence on demand, rent, and the prices of both house types. Focusing on the rent, it is explicitly modeled in the no arbitrage condition from Equation 7. The rent actually measures rent-energy combined for both brown and green houses.

[12]Gains and costs from retrofit are uncertain and volatile, thus the cost of an effective retrofitting is very difficult to assess. According to the report by Pisany-Ferry and Mahfouz (2023), the average cost of retrofitting is between 20k€ and 37k€. However, report from the University of Notthingham (2022) points the underestimating cost of retrofitting, especially "deep retrofits", that would be on average of 69k£, i.e. approximatively 79.7k€. The baseline calibration is closer to Pisany-Ferry and Mahfouz (2023). However, one could consider that the calibration underestimates the total cost, as it is assumed that the supposed utility cost of retrofitting is taken into account in the functional form of the monetary cost.

A certain body of literature empirically assesses the role of differential energy costs in rents between highly and poorly performing residences. The advantages of renting a highly energy-efficient house extend beyond reduced rent payments for tenants; they also encompass a diminished sensitivity to fluctuations in energy prices and weather conditions, which impact energy bills. Consequently, this term "energy price" is not precisely reflective of a true energy cost. The challenge lies in determining whether the observed green premium stems from information about certification or from the landlord's enhanced energy awareness. This is why it designates more a "green premium.". For example, Cajias and Piazolo (2012) indicate that a one percent increase in energy consumption corresponds to a 0.075% decrease in rents in Germany. Furthermore, a detailed multi-region analysis by the European Commission European Commission. Directorate General for Energy. et al. (2019) discovers that energy efficiency improvements correlate with a 4.4% rent increase in Austria (for instance, moving from a 'D' rating to a 'C' rating). In Belgium, Bala et al. (2014) demonstrate that rents rise in properties equipped with double glazing and wall insulation.

The calibration's objective is also to model the (quantitatively) minor difference between owning a brown and a green house. According to INSEE (2019), the average French household dedicates 5 percent of its yearly expenditures to energy. Our aim is not to precisely calibrate the European energy efficiency label due to our focus on carbon emissions, which also encompasses the type of energy used. We set $p_e = 0.0075$, which yields a differential of approximately 400€ per year for one house, following the results of Astier et al. (2024) about the effective energy consumption by energy efficiency level.

4.2 Moments of the calibrated model

Before proceeding to the counterfactual policy analysis, we validate the model's ability to replicate salient features of the French housing market. Table 2 reports the model's fit to key aggregate moments, including the homeownership rate and the green price premium. Figure 2 demonstrates that the calibrated economy successfully reproduces the cross-sectional distribution of home ownership. Finally, Figure 3 characterizes the baseline steady-state sorting of households across tenure status and energy efficiency levels.

Aggregate moments Table 2 presents a comparison between the model's results and the targeted values for several key housing metrics. The model predicts a home-ownership rate of 58.7%, which is slightly lower than the targeted rate of 59%. Regarding housing characteristics, the model estimates that 35.8% of homes are brown (classified as EPC E-G), while the target is 35% in 2021.

	Model	Data	Source
Targeted moments			
Homeownership rate	58.7%	59%	Insee (2021)
Landlordship rate	14.2%	13%	Insee (2021)
Share of brown houses (EPC E-G)	34.3%	35%	SDES (2021)
Untargeted moments			
Green to brown price ratio	1.0325	1.0027	SeLoger (2021)
Expenditure share on housing (renter)	24.9%	28.5%	Insee (2021)
Share of brown housing retrofitted	0.31%	0.22%	ONRE (2021)

Table 2: Aggregate Moments

Regarding the untargeted moments, the green to brown price ratio (green premium) in the model is 1.0325, which is much higher than in the data from the website SeLoger, where the green premium is around 1.0027. For renters, the model suggests an expenditure share of 24.9% on housing, which is lower than the target of 28.6%. Finally, we overestimate the yearly share of retrofits, at 0.31% in the model compared to 0.22% of effective major retrofits in the data.

Housing distribution The French housing market has an unequal housing distribution: around a quarter of households own more than one house, 13% are landlords and only 3.5% of households own 50% of the rental dwelling stock. A main objective of the calibration is to replicate this degree of concentration in the model, as it is a key object for the relevance of the quantitative results. Figure 2 shows the model's performance at matching the residential housing distribution. However, we don't replicate enough the concentration of the supply of housing: only 26% of the rental services are supplied by the multiple owners of more than 5 houses (compared to 50% in the data).

Tenure distribution Figure 3 presents the baseline economy, offering a detailed breakdown of households across three key dimensions: income, housing status (tenure) and housing energy efficiency. First, households are separated between three income categories: low, middle and high income[13]. Within each income category, the Figure displays the housing distribution

[13]There are seven discrete states of income. Low income households denote the income categories between 1 and 3, which represent 33.1 percent of households. Middle income households include the income category 4, which represents 33.7 percent of households. And high income denote the rest, with the income category between 5 and 7. Middle income households receive the median and average wage of the economy, which is $w = 1$.

of households, providing information on the housing status (renter, owner-occupier, landlord) and the housing energy efficiency. The y-axis measures the mass of household, within an income category, corresponding to the specific housing status and color. Thus, the sum of the bars in an income category equals 100%.

In line with the data, there is a notable downward trend in the proportion of renters as income levels increase. On the opposite, landlords are more likely to be high income households. The baseline economy does not depict a clear gradient in terms of sorting of households on the houses energy efficiency, which. Interestingly, the model predicts a higher green share among landlords than owner-occupiers, because green landlordship is associated with a higher rental income. This results from the assumption on the renters' no arbitrage condition, which creates a full pass through of the energy premium to rental prices.

5 Policy Analysis

In this section, we compare the macroeconomic and distributional effects of three climate policies in the housing market: a tax on energy, a tax on brown rental income and a retrofit subsidy. We first analyze how the policies impact the prices and main aggregate outcomes such as the homeownership rate. Then, we focus on the change in the housing allocation (by status) across income levels.

5.1 Policy experiments

We consider three climate policy instruments. The first is an energy tax, similar in spirit to a carbon tax but more generally capturing an increase in energy prices—such as the structural

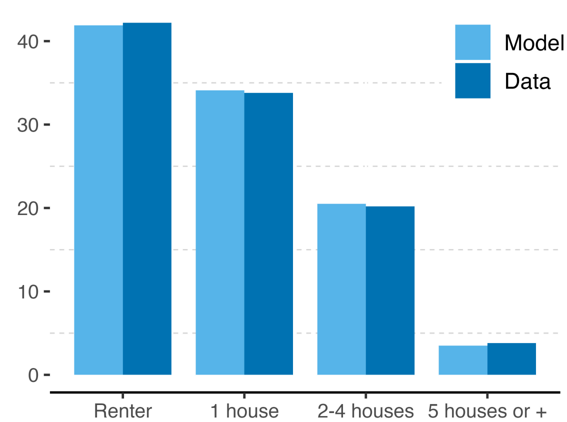

Figure 2: Moment matching: Housing distribution

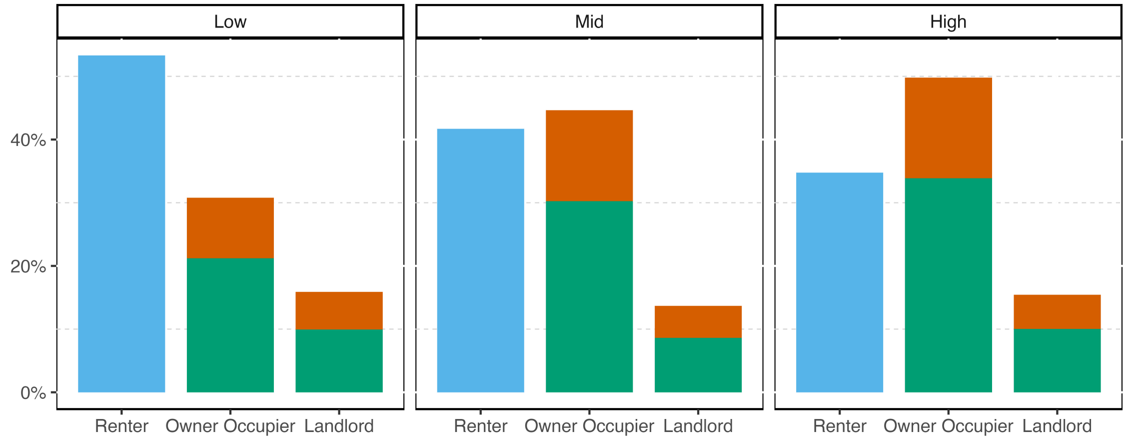

Figure 3: Initial steady state - Housing status by income

rise observed after the war in Ukraine.[14] This policy raises the user cost of living in a brown house for both renters and owners, affecting the 45% of households who occupy brown units in our baseline calibration. The second policy is a tax on brown rental income, a less common but relevant instrument for the housing market.[15] It directly targets brown landlords—who represent 5% of households in our initial calibration. This avoids placing the burden directly on renters while reducing the profitability of renting out brown units. The third instrument is a retrofit subsidy, which lowers the cost of upgrading brown homes to green. This policy directly benefits the 18% of households who own brown properties and can choose to retrofit.

All three policies are calibrated to achieve the same target level of green housing (75%) and are required to be budget-neutral.

5.2 Prices and aggregate outcomes

Table 3 reports the results on prices and aggregate outcomes of the baseline equilibrium presented in Section 4 and the three policies experiments. The Table reports the green and brown prices and their ratio, denoted as the *green premium*. It includes the green rent only as it measures the rental expenditures for renters and the prices to rent ratio. Then, the distribution of households is reported between the five housing status: renter, green and brown owner and landlord. Finally, we include the share of households retrofitting at steady state.

All policies are budget neutral and targeted to match a green housing share of 75%.

[14] For instance, permanent shifts in energy markets following the Ukraine war have led to sustained increases in energy prices.

[15] It is directly inspired by a regulation implemented in France that prohibits the rental of the least energy-efficient dwellings, effectively requiring landlords to undertake retrofits.

	Baseline	Tax on energy	Tax on brown rental income	Retrofit subsidy
Green share (%)	65.2	75.0	75.0	75.0
Prices				
Green premium (q_g/q_b)	1.030	1.052	1.047	1.023
Brown house price (q_b)	2.541	2.495	2.519	2.551
Green house price (q_g)	2.618	2.626	2.638	2.609
Brown price to rent ratio (q_b/ρ_g)	10.374	10.153	10.119	10.405
Green price to rent ratio (q_g/ρ_g)	10.689	10.684	10.597	10.641
Green rent (ρ_g)	0.245	0.246	0.249	0.245
Household shares				
Renters (%)	41.35	41.53	40.32	41.15
Brown Owner-occupiers (%)	12.53	9.23	13.85	9.75
Green Owner-occupiers (%)	26.79	29.84	26.95	29.86
Brown Landlords (%)	5.19	3.82	3.12	3.67
Green Landlords (%)	9.03	10.45	10.69	10.50
Retrofitters (%)	0.31	0.45	0.45	0.47

Table 3: Policy comparison - Prices and aggregates

Prices The main effects of the policies is captured by the green premium, e.g. the green to brown price ratio. It decreases under the subsidy and increases under the two taxes. This difference comes from the nature of the policies: the subsidy decreases the cost of retrofitting, reducing the gap between brown and green housing. The two taxes create penalty in being a brown user/landlord, increasing the incentives to becoming green and therefore the gap between the two prices. The absolute prices have the similar changes: the two taxes decrease the brown house price and increase the green house price, while the retrofit subsidy does the opposite. The taxes attain the objective of the green housing share by making the green houses more *desirable* while the subsidy makes the subsidy make green housing more *affordable* through retrofitting.

Regarding the impact on the rent, it remains roughly stable with the tax on energy and the subsidy, but it increases with the tax on brown rental income, as rental supply is more scarce.

Housing allocation The evolution of the renter share captures the main effects of the three policies: it increases with the tax on energy (access to both brown and green homeownership has become more difficult), it decreases with the tax on brown rental income (as the rent has increased) and it decreases under the retrofit subsidy (green affordability). Under the tax on energy, despite the increase in the rent and the decrease in the brown price to rent ratio, the renter share has increased (+0.18 pp). This is due to the fact that even though the brown house price has become more affordable, the user cost of living in a brown house has increased, relatively favoring the renter cost.

One striking effect is the increase (+1.32 pp) in the share of brown owner occupiers under the tax on brown rental income, where it has largely decreased with the two other policies. This can come from either former landlords, preferring to be only owner occupier due to the decrease in their rental income or from former renters, who prefer not to pay a higher rent and benefit from the decrease in brown house price.

The retrofit subsidy increases the most the green owner share (owner occupier and landlord), reflecting the higher affordability of green housing.

Finally, we can notice that all policies increase the share of retrofitters (i.e. household retrofitting their house from brown to green), but the retrofit subsidy is favoring it the most.

5.3 Distributional outcomes

The second part of the policy comparison focus on the distributional outcomes. In particular, we analyze how the policies have impacted the housing allocation by income. The Figures 4, 5 and 6 plot the variation in the housing allocation for each policy. Each figure represents the percentage point of difference between the baseline scenario (Figure 3) and the policy experiments, in terms of variation share within income categories.

Tax on energy In order to reach the 75.0 % green share, the tax on energy raises the energy cost differential by 111.3%. The tax increases the user cost of brown housing services.

Moreover, for a given green rent, it decreases the brown rent, thus decreasing the rental income of brown landlords, through the no arbitrage condition (Equation 7). As the cost of using a brown house increases, its price decreases and a shift of the demand is observed for green houses and rental services.

Figure 4 shows a mirror effect suggesting a strong substitution between brown and green, quite homogeneous across income levels and housing status. However, we can observe that the increase in the renter share is driven almost entirely by the low and middle income households, who don't fully substitute between brown and green. Despite the increase in the rental equilibrium, brown homeownership is less viable (it is more costly to live in a brown house) and green homeownership is less affordable (it is more costly to buy a green house). Therefore, a part of the low and middle income cannot subsitute towards green homeownership

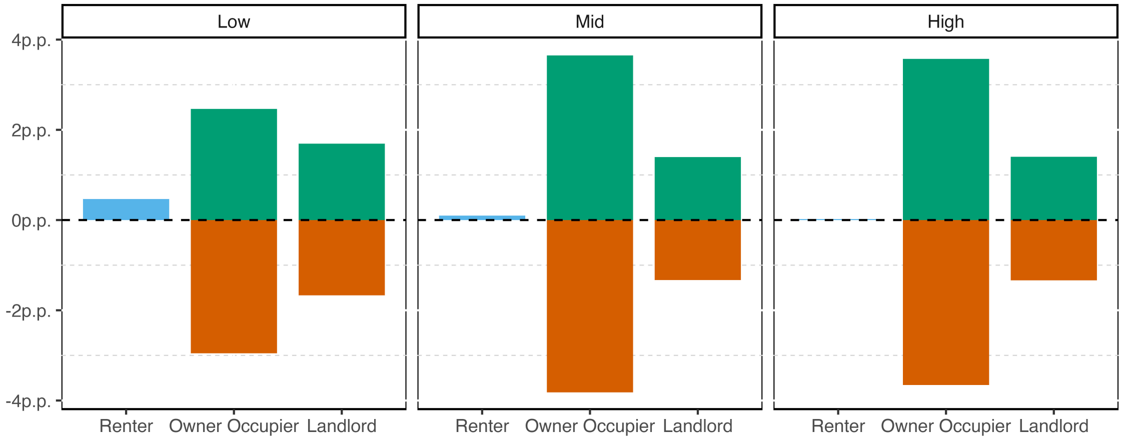

Figure 4: Tax on energy - Housing status by income

and become renters.

Tax on brown rental income In order to reach the 75.0 % green share, the tax on brown rental income is set to 10.3%. The tax increases the equilibrium rent and the green premium.

The primary policy's effect is a direct reduction in the proportion of brown landlords, for all income levels, only partially offset by an uptick in the number of green landlords (Figure 5). We do observe a decrease in green owner occupiers for low and middle income, suggesting that some choose to become green landlords as it has become more profitable thanks to the increase in the equilibrium rent. Despite these substitutions, there is a decrease in the overall share of landlords, reducing the rental services supply.

Across all income groups, the renter share decreases. For the high income, this results in a strong increase in green home-ownership, for both owner occupiers and landlords. It is different for the low and middle income, where the green landlord share increases as well as the brown owner occupier share. Unlike for the energy tax, the brown house price has decreased but the user cost of living in a brown house has remained the same, as the policy only targets landlords' income. This yields to a brown housing reallocation driven by the low and middle income: renting is more expensive, green ownership is less accessible, then brown ownership becomes a relatively more favored option. Indeed, in terms of incentives, living in a brown house as an owner occupier has become more desirable relative to be a renter and a green owner.

Retrofit Subsidy In order to reach the 75.0 % green share, the retrofit subsidy is set to 18.2%. Compared to the other two policies, the effects on housing allocation appear quite homoge-

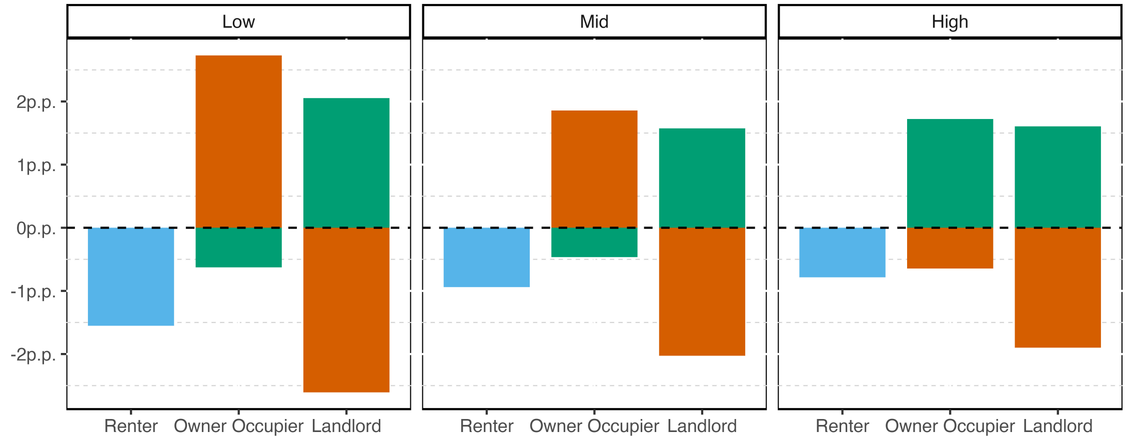

Figure 5: Tax on brown rental income - Housing status by income

neous across income. We observe a strong substitution between brown and green housing, for both owner occupiers and landlords, and a small decrease in the renter share.

The distribution across housing status remains stable, suggesting that the shift toward green housing has not been driven by market transactions but rather by direct retrofitting (Figure 6. Essentially, the policy facilitates an increase in the green housing share without significantly altering housing affordability or ownership structures. Consequently, the subsidy primarily operates at the margin, enabling existing brown properties to convert into green properties without disrupting the broader housing market.

Furthermore, the decline in the renter share cannot be attributed to rising rents. Instead, it appears to stem from a reduction in green house prices, which improves the affordability of green units and allows more households to transition out of renting and into ownership. Finally, the subsidy is the only policy that yields to quite homogeneous effects across income levels.

6 Perspectives

We aim to enrich the model with a few extensions and to improve the policy analysis. We detail below what we plan to include to the paper.

6.1 Extensions

Retrofit when buying or selling According to a recent survey in France (ONRE, 2021), 43% of major retrofits are undertaken at the time of a home purchase. A similar pattern emerges

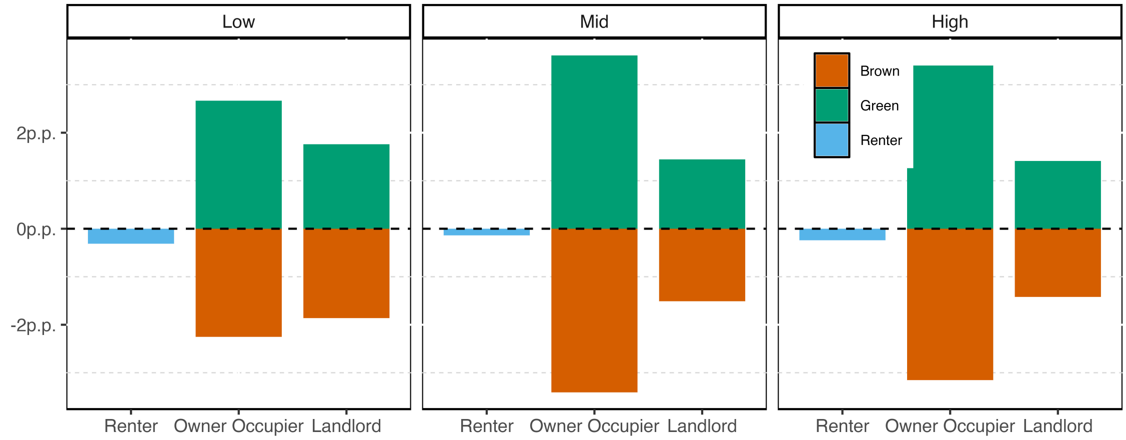

Figure 6: Retrofit Subsidy - Housing status by income

in the Belgian region of Wallonia, where new regulations require households to retrofit upon buying a property. Conversely, some households may live in energy-inefficient dwellings yet choose to retrofit before selling in order to capture the green premium. These behaviors reflect the fact that housing transactions—whether buying or selling—are moments when households are more likely to undertake renovation work, whether energy-related or not. As a result, energy retrofits become easier and more frequent during these periods.

To capture these dynamics, the model could incorporate a different utility cost of retrofitting at the time of a transaction. This would improve its realism and introduce an additional mechanism linking retrofitting behavior to housing market liquidity.

Landlord-tenant dilemma One important limitation of our modeling framework is the weak misalignment of incentives between renters and landlords. In the current setup, landlords and tenants occupy housing of the same "color" meaning that any retrofit decision affects both parties symmetrically. This largely eliminates the split-incentive problem extensively discussed in the environmental economics literature (e.g., Charlier and Kahouli (2019)).

We could allow households to own houses of different colors, where landlords could choose to live in a green house and to rent their brown house.

Fuel poverty assessment The model could be expanded to include fuel poverty. This would introduce energy as a choice variable, where households choose to "underheat" if they are financially constrained. This adds another dimension to the welfare effects of retrofitting, by limiting the number of fuel poor households. Moreover, this would also bring to the analysis a partial "rebound effect", as the energy reduction from the retrofitting might be lower than expected due to the initial underheating. In our setting, this would break the no arbitrage condition on the rental market and improve the effects of the energy price path through to rent.

6.2 Policy analysis

Transition analysis A transitional analysis would provide a more comprehensive assessment, as policy objectives ultimately concern the speed of the transition – how quickly a target green share can be reached. In this setting, we could model a ban on brown investment, similar to recent measures implemented in France, and compare how different policies affect the pace at which the brown housing stock declines beyond depreciation. This decline would be driven either by retrofitting or by an increase in green construction. Moreover, incorporating transition dynamics would enable us to examine the distributional costs that arise over time and to conduct a full welfare analysis.

Mean-tested subsidies Drawing on the example of France and the insights from the environmental economics literature, means-tested subsidies may offer a fairer and more efficient alternative to un-targetted subsidies. This suggests that the model could incorporate a more detailed subsidy schedule that varies with household income.

Policy mix Finally, we could assess how the policies interact within a policy mix, evaluating whether they operate as substitutes or complements.

7 Conclusion

This study develops a framework for evaluating the macroeconomic and distributional impacts of climate policies in the housing market. We provide new insights into how these policies affect the house prices and the different groups—renters, homeowners, and landlords—across income levels. We find that the tax on energy and the tax on brown rental income increase the green premium, while the subsidy decreases it. Therefore, the subsidy, financed with an increase in the income tax, favors the most the access to green homeownership.

The tax on energy and the tax on brown rental income rely on changing the incentives to increase the green share. This reflects in the market prices, where the brown house price decreases, while the green house price increases, becoming less affordable. However, they both affect the housing market in a different way. The tax on energy reduces access to homeownership as brown houses are more costly to live in, and green houses are more costly to buy. This results in an increase of the renter share. The tax on brown rental income increases the green share driven by the increase in the green landlords. However, the rent equilibrium has increased, decreasing the renter share. As brown housing is cheaper and the user cost is unchanged, former renters reallocate to become brown owner occupiers.

The perspectives described in the previous section will allow a more complete policy analysis to further understand the macroeconomic and distributional implications of the climate policies along the transition.

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A Algorithm

This section describes the main elements of the algorithm we use to solve the quantitative heterogeneous agent model of the housing market presented in the paper.

We compute the stationary policy functions by exploiting the recursive structure of the household problem. The simultaneous presence of discrete housing adjustments—such as tenure choice and retrofitting—and continuous savings choices renders the optimization problem non-convex. Consequently, the value function exhibits kinks, and the Euler equation provides a necessary but not sufficient condition for optimality. To address this challenge, we employ the Nested Endogenous Grid Method (NEGM) developed by Druedahl (2021). This algorithm decomposes the problem into a continuous consumption-savings block and a discrete housing choice block. We solve the continuous block using the Endogenous Grid Method combined with an upper envelope scan to discard suboptimal local maxima. We smooth the discrete choice step by introducing extreme value taste shocks, following Iskhakov et al. (2017).

We solve the distribution law of motion using standard methods in the HA literature, and clear markets using a quasi-Newton solver.

A.1 Feasibility correspondence

We first summarize with two figures the main discrete choices that household face, namely inter-temporal discrete choices on the housing stock, and intra-temporal discrete choices on landlordship and housing services.

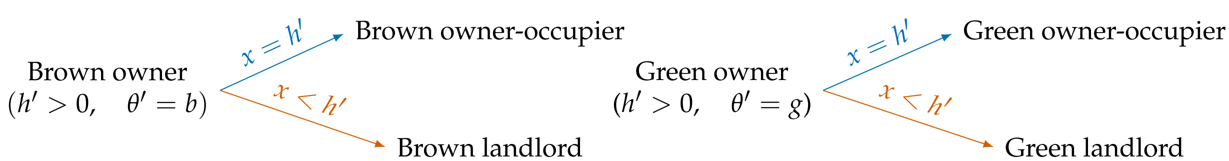

Figure 8: Intra-temporal discrete choice on landlorship.

A.2 Stationary solution to the household problem

Household problem We solve for the fixed point of the Bellman Equation describing the household problem (Equation 5). We split the household problem into three main stages, and solve the model using backward induction, and apply iteratively the Bellman operator,

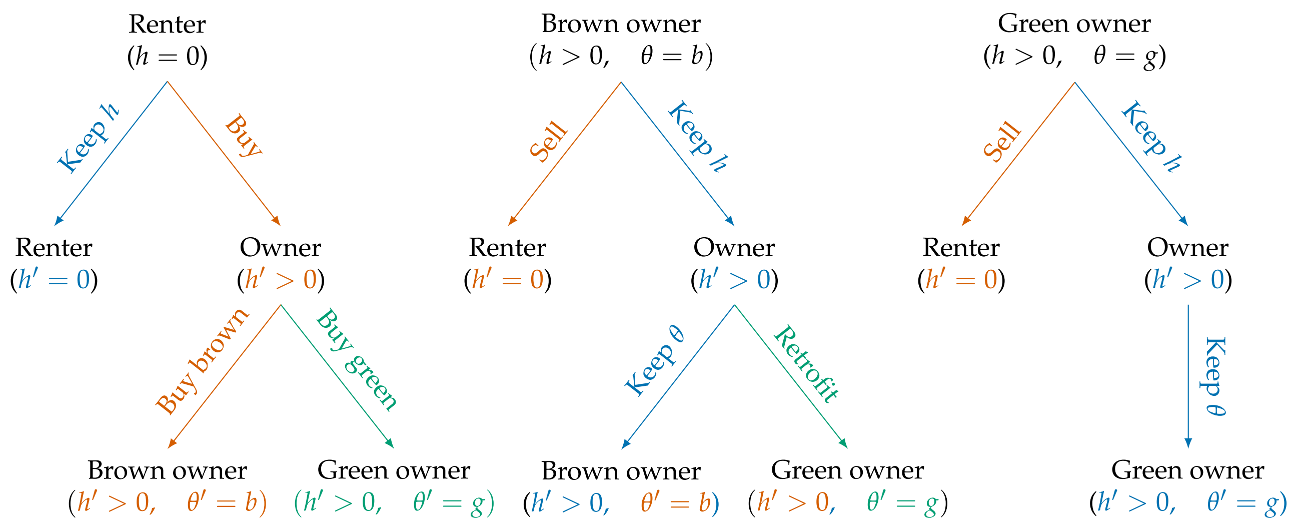

Figure 7: Inter-temporal discrete choices

starting from an initial guess on the value function. At iteration i, starting from a candidate value function $V^{(i-1)}$, we apply:

$$V^{(i)}(s,a,h,\theta) = \underbrace{\max_{x,h',\theta'}}_{\textbf{stage 1}} \underbrace{\left\{ \max_{c,a} \left\{ u(c,x) + \underbrace{\beta \times \mathbb{E}V^{(i-1)}(s',a',h',\theta')}_{\textbf{stage 3}} \right\} \right\}}_{\textbf{stage 2}}$$

$$\text{s.t} \qquad \text{Budget constraint and feasibility hold at each stage}$$

Until $||V^{(i)}, V^{(i-1)}||_\infty < 10^{-9}$.

Fitted Value Function Iteration We fit the state space on a meshed grid, and operate on it. We let the choice on savings be off-grid, and use linear interpolation when needed. The grid on income ($\mathcal{G}_s$) is given by the quadrature method from Tauchen (1986), the asset grid ($\mathcal{A}$) is built as a double exponential grid, and the grids on h, θ correspond to the housing structure in the economy (discrete choices).

Inputs We solve for the household problem given aggregate housing prices and rent, exogenous climate policies and a proportional income tax. Prices and policies give us the components of the budget constraint. We take two initial guesses to start the iterative algorithm:

- Initial guess on the value function $V^{(0)}(s,a,h,\theta)$. We set it to zero.
- Initial guess on the marginal value of assets $\partial V^{(0)}(s,a,h,\theta)/\partial a$. We set it to zero, consistently with the guess on the value function.

We also pre-compute the feasibility correspondences on discrete choices (Figure 7), and store the feasible indexes in matrices denoted $\mathcal{F}_y(z)$, where y is the choice of interest and z the state (or the control) that influences feasibility.

Stage 3: Continuation value & stochastic depreciation

First, we follow Druedahl (2021) and pre-compute for any post-decision state $(s,a',\tilde{h}',\theta')$ the continuation value – applying the expectation operator on the income state, and on next period's housing stock (stochastic depreciation), where $\tilde{h}'$ denotes the housing stock choice of the agent before the exogenous depreciation shock. It yields the following post-decision continuation value function $\mathcal{W}^{(i)}(s,a',\tilde{h}',\theta')$

$$\mathcal{W}^{(i)}(s,a',\tilde{h}',\theta') = \beta \sum_{\forall s'|s} \pi_{s,s'} \times \begin{cases} \Big[(1-\Delta)V(s',a',\tilde{h}',\theta') + \underbrace{\Delta V(s',a',h'_{-1},\theta')}_{\text{Stochastic depreciation}}\Big] \\ V(s',a',h',\theta') & \text{if} \quad h'=0 \end{cases}$$

The same operation is applied to the marginal value of savings, to get $\partial_a \mathcal{W}^{(i)}(s,a',\tilde{h}',\theta')$.

Stage 2: *Conditional* consumption-saving choice

Then, we solve for the *choice specific* consumption-saving program, that is continuous choice policy functions, conditional on the discrete choice decisions (stage 1). The relevant states at this stage are $(s,a,h,\theta,h',\theta',x)$ or equivalently $(s,l,\tilde{h}',\theta',x)$, where l is the post-discrete-choices cash-on-hands. It takes values from Ψ, a tensor that subtracts the begin-of-period cash on hands state space to the net monetary cost of any feasible discrete decisions. It maps $\omega \times (\tilde{h}',\theta',x) \mapsto \mathbb{R}$.

$$\mathcal{V}^{(i)}(s,l,\tilde{h}',\theta',x) \quad = \begin{cases} \max_{a'} u(c,x) + \mathcal{W}^{(i)}(s,a,\tilde{h}',\theta',x) & \forall x \in \mathcal{F}_x(\tilde{h}') \\ -\infty & \text{otherwise} \end{cases}$$
$$\text{s.t} \quad c = l - a \qquad \text{and} \quad l \in \mathcal{L}^{\text{exo}} \equiv \Psi(\omega,\tilde{h}',\theta',x)$$

Where $x \in \mathcal{F}_x(\tilde{h}')$ accounts for the feasibility in the choice of x, that is constrained by the choice on housing stock (renters may choose any $x \in \mathcal{G}_x$, but owners $\tilde{h}' > 0$ can only choose $x \leq \tilde{h}'$).

The Euler Equation corresponding to this program states that conditional on x, the marginal utility function of consumption should be equal to the marginal value of saving on post-decision states.

$$\frac{\partial u}{\partial c}(c, x) = \frac{\partial \mathcal{W}^{(i)}}{\partial a'}(s, a', \tilde{h}', \theta')$$

From the envelope theorem, the marginal value of saving is equal to the discounted marginal value of cash tomorrow, such that the marginal value of saving is an immediate function of the marginal utility of consuming tomorrow. We use this condition to update the marginal value of savings for the next iteration (Druedahl and Jørgensen, 2017; Iskhakov et al., 2017; Gaillard, 2020; Druedahl, 2021).

$$\frac{\partial \mathcal{V}^{(i)}}{\partial a} = (1 + r) \times \frac{\partial u}{\partial c^*}$$

DC-EGM Algorithm The problem is non-convex, as the continuation value may be kinked from discrete decisions occurring at the past iterations. Therefore, the Euler Equation is a necessary but insufficient condition to optimality.[16] We follow Iskhakov et al. (2017) and build an endogenous grid method coupled with an upper-envelope step (DC-EGM) on the *conditional* value function to find the *conditional* policy functions of this problem. In particular, the algorithm does the following

1. **Inputs** The EGM step takes as inputs two elements:
   - The continuation value $\mathcal{W}(s, a, \tilde{h}', \theta', x)$, that is importantly a function of the agents' savings at the current period.
   - The marginal value of savings $q \equiv \frac{\partial \mathcal{W}}{\partial a'}$, which is extracted from the stage 3.
2. **Initialization** We create an exogenous grid of savings, that must be consistent with the borrowing constraint, and with the grid on cash-on-hands. We initialize $\mathcal{V}$ as a matrix filled with $-\infty$.

[16] At a given point $(s, l, \tilde{h}, \theta', x)$, there might be multiple roots to the Euler Equation, but it remains possible to select the roots that yield the highest $\mathcal{V}$ by taking an envelope (Iskhakov et al., 2017).

3. **Euler Equation & Endogenous grid** Take the exogenous grid on savings (a'), we extract the implied consumption choice by the Euler Equation. Denote by z the inverse marginal utility function, such that $z(.|x) = \{\frac{\partial u}{\partial c}(.|x)\}^{-1}$. The Euler Equation states:

$$c(a') = z\Big(q(a') \quad | \quad x\Big) \qquad \forall a' \in \mathcal{A} \quad \text{, conditional on } s, \tilde{h}', \theta', x$$

The budget constraint yields an endogenous grid on the mid-period cash-on-hands.

$$\mathcal{L}^{\text{endo}} = \mathcal{A}' + \mathbf{c} \qquad \text{where} \quad \mathbf{c} = c(a') \quad \forall a' \in \mathcal{A} \qquad \text{, conditional on } s, \tilde{h}', \theta', x$$

4. **Borrowing Constraint** When the household endogenously chooses to locate at the borrowing constraint, the Euler Equation does not hold with equality. We identify these households since they exhibit an exogenous cash-on-hand inferior to the cash-on-hands endogenous to the first point of the exogenous saving grid. We force consumption to be equal to cash-on-hands, augmented by the borrowing constraint and compute the associated intermediate value function.

$$\text{If} \quad l \leq \mathcal{L}_1^{\text{endo}}, \quad \text{then} \quad \begin{cases} \widetilde{\text{cpol}}\Big(s, l, \tilde{h}', \theta', x\Big) = l - \mathcal{A}'_1 \\ \mathcal{V}(s, l, \tilde{h}', \theta', x) = u\Big(\widetilde{\text{cpol}}(s, l, \tilde{h}', \theta', x), x\Big) \quad + \quad \mathcal{W}\Big(s, \mathcal{A}'_1, \tilde{h}', \theta'\Big) \end{cases}$$

5. **Upper Envelope Step** We implement an upper-envelope algorithm to discard the points that are not optimal to find the optimal consumption policy function, described bellow

   (a) We loop on the exogenous saving grid $\mathcal{A}$ and store the index as `ia`

   (b) We loop on the exogenous cash-on-hands grid $\mathcal{L}^{\text{exo}}$ and store the index as `il`

   (c) If $\mathcal{L}^{\text{exo}}_{\texttt{il}} \in [\mathcal{L}^{\text{endo}}_{\texttt{ia}}, \mathcal{L}^{\text{endo}}_{\texttt{ia+1}}]$, we interpolate the consumption rule and the intermediate value function at $\mathcal{L}^{\text{exo}}_{\texttt{il}}$ using the inverted Euler Equation. Formally, conditional on $s', \tilde{h}', \theta', x$:

$$c_{\texttt{il,ia}} = \mathbf{c}_{\texttt{ia}} + \frac{\mathbf{c}_{\texttt{ia+1}} - \mathbf{c}_{\texttt{ia}}}{\mathcal{L}^{\text{endo}}_{\texttt{ia+1}} - \mathcal{L}^{\text{endo}}_{\texttt{ia}}} \times \left(\mathcal{L}^{\text{exo}}_{\texttt{il}} - \mathcal{L}^{\text{endo}}_{\texttt{ia}}\right)$$

$$\widetilde{\mathcal{V}_{\texttt{il,ia}}} = u\Big(c_{\texttt{il,ia}}, x\Big) + \mathcal{W}\Big(\mathcal{A}_{\texttt{ia}}\Big) + \frac{\mathcal{W}\Big(\mathcal{A}_{\texttt{ia+1}}\Big) - \mathcal{W}\Big(\mathcal{A}_{\texttt{ia}}\Big)}{\mathcal{L}^{\text{endo}}_{\texttt{ia+1}} - \mathcal{L}^{\text{endo}}_{\texttt{ia}}} \times \Big((\mathcal{L}^{\text{endo}}_{\texttt{ia}} - c_{\text{il,ia}}) - \mathcal{A}_{\texttt{ia}}\Big)$$

   (d) If $\widetilde{\mathcal{V}_{\texttt{il,ia}}}$ is the best candidate value yet found for the grid-point $\mathcal{L}^{\text{exo}}_{\texttt{il}}$, we store it in $\mathcal{V}$ and the conditional consumption policy function in $\widetilde{\text{cpol}}$.

The algorithm outputs the resulting $\mathcal{V}^{(i)}$ on the space $\omega \times (\tilde{h}', \theta', x)$. When the underlying discrete choice yielded a cash-on-hands level l lower than the borrowing constraint, we penalize $\mathcal{V}^{(i)}$ to $-\infty$, so this choice will never be optimal in the first stage.

Stage 1: Discrete Choice & Taste shocks

Taste shocks The last step of the problem is to maximize the *conditional* value function on the feasible discrete decisions, at any point of the state space. We follow Iskhakov et al. (2017) and introduce Extreme Value taste shocks, drawn from a Gumbel distribution, on any discrete decision the agent faces.

These shocks play the role of logit smoothers on the discrete decisions (Iskhakov et al., 2017; Druedahl and Jørgensen, 2017). They have multiple benefits: (1) since they smooth discrete decisions into probabilities, they ease equilibrium finding and prevent bang-bang price iterations, (2) they reduce the kinks on the stationary value function, allowing to control the dimensionality on the asset grid without incuring a dramatic loss in precision, (3) they allow to build Jacobians to compute the transition (Auclert, Bardóczy, Rognlie, and Straub, 2021). To control the degree on smoothing on each decisions, we introduce a sequence of preference shocks that respects the restriction of the feasibility set. The variance of these shocks constitute additional parameters that we use in the internal calibration to match cross-sectional moments.

We convert the following discrete choice problem to a soft-max formula.[17]

$$
\begin{aligned}
V^{(i)}(s,a,h,\theta) \quad &= \quad \max_{\tilde{h}',\theta',x} \left\{ \mathcal{V}^{(i)}(s,a,h,\theta,\tilde{h}',\theta',x) \right\} \\
\text{s.t} \quad \forall\ \tilde{h}' \in \mathcal{F}_{\tilde{h}'}(h) \quad &, \quad \theta' \in \mathcal{F}_{\theta'}(\tilde{h}',\theta), \quad x \in \mathcal{F}_x(\tilde{h}')
\end{aligned}
$$

Since the choice variables at that stage are discrete, this problem can be split into three steps.

Stage 1.3: Dwelling & housing stock choices We first find the optimal housing service and housing stock choices, conditional on the housing status ($j' \in$ {renter, owner-occupier, landlord}) and the type of housing. We use the conditional value function to find the optimal dwelling

[17] Interested readers can find the proof in Iskhakov et al. (2017)

and housing stock choices.

$$\begin{aligned} v_{\text{dwelling}}(\omega, j', \theta') &= \mathbb{E}_{\varepsilon_{h',x}} \left(\max_{h',x} \left\{ \mathcal{V}^{(i)}(\omega, \tilde{h}', \theta', x) + \nu_{\varepsilon_{\tilde{h}',x}} \varepsilon_{\tilde{h}',x} \right\} \right) \\ \text{s.t} &\quad \tilde{h}' \in \mathcal{F}_{\tilde{h}'}(h, j') \quad \text{and} \quad x \in \mathcal{F}_x(\tilde{h}', j') \end{aligned}$$

Where $\varepsilon_{\tilde{h}',x}$ are Gumbel shocks with variance $\nu_{\tilde{h}',x}$. Since the Gumbel shocks are idependently drawn, one can use the log-sum formula to compute the value function, and the associated probabilities to choose the dwelling and housing stock. For for feasible $\tilde{h}', x, \theta', j' | \omega$:

$$\begin{aligned} v_{\text{dwelling}}(\omega, j', \theta') &= \nu_{\varepsilon_{\tilde{h}',x}} \log \left[\sum_{\tilde{h}' \in \mathcal{F}_{\tilde{h}'}(h,j'), x \in \mathcal{F}_x(\tilde{h}',j')} \exp\left(\frac{V_{\text{candidate}}(\omega, \tilde{h}', \theta', x)}{\nu_{\varepsilon_{\tilde{h}',x}}} \right) \right] \\ \mathbb{P}(\tilde{h}', x | \{\omega\}, j', \theta') &= \frac{\exp\left(V_{\text{candidate}}(\omega, \tilde{h}', \theta', x) \Big/ \nu_{\varepsilon_{\tilde{h}',x}} \right)}{\sum_{\tilde{h}' \in \mathcal{F}_{\tilde{h}'}(h,j'), x \in \mathcal{F}_x(\tilde{h}',j')} \exp\left(V_{\text{candidate}}(\omega, \tilde{h}', \theta', x) \Big/ \nu_{\varepsilon_{\tilde{h}',x}} \right)} \end{aligned}$$

The intermediate value obtained is multi-dimensional: it has the same dimensions as the state space, augmented by the sizes of j', θ'. On the other hand, the probabilities of choosing each dwelling and housing stock are computed for each feasible transition, which implies that they are stored in a 8-D tensor.

Stage 1.2: House color choice Then, we find the optimal housing type choice, conditional on the housing status. We add Gumbel shocks on each alternative with variance $\nu_{\theta'}$. We use the previously computed value function to get the following result.

$$\begin{aligned} v_{\text{color}}(\omega, j') &= \nu_{\theta'} \log \left[\sum_{\theta' \in \mathcal{F}_{\theta'}(j')} \exp\left(\frac{v_{\text{dwelling}}(\omega, j', \theta')}{\nu_{\theta'}} \right) \right] \qquad \text{for} \quad j' \in \mathbb{J} \\ \mathbb{P}(\theta' | \{\omega\}, j') &= \frac{\exp\left(v_{\text{dwelling}}(\omega, j', \theta') \Big/ \nu_{\theta'} \right)}{\sum_{\theta'} \exp\left(v_{\text{dwelling}}(\omega, j', \theta') \Big/ \nu_{\theta'} \right)} \qquad \text{for} \quad \theta' \in \mathcal{F}_{\theta'}(j', \theta) \quad \& \quad j' \in \mathbb{J} \end{aligned}$$

Here, feasibility depends on θ because of the asymmetry of the choice on green owners.

Stage 1.1: Housing tenure choice The last step is to find the optimal decision rule of the household between being a renter, an owner-occupier and a landlord, conditional on the current housing states. Doing so, we find the overall *expected* value function of the household.

$$V^{(i)}(s,a,h,\theta) \quad = \quad \nu_{\varepsilon_{j'}} \log\left[\sum_{j'\in\mathbb{J}} \exp\left(\frac{v_{\text{color}}(\omega,j')}{\nu_{\varepsilon_{j'}}}\right)\right]$$

Together with the probability of choosing any housing status, conditional on the state space.

$$\mathbb{P}(j'|\omega) \quad = \quad \frac{\exp\left(v_{\text{color}}(\omega,j')\Big/\nu_{\varepsilon_{j'}}\right)}{\sum_{j'}\exp\left(v_{\text{color}}(\omega,j')\Big/\nu_{\varepsilon_{j'}}\right)} \qquad \text{for} \quad j'\in\mathbb{J}$$

Discrete choice probabilistic policy functions The conditional probabilities $\mathbb{P}(j'|\omega)$, $\mathbb{P}(\theta'|\{\omega\},j')$ and $\mathbb{P}(\tilde{h}',x|\{\omega\},j',\theta')$ can be multiplied to get the total probability to choose $(\tilde{h}',\theta',x)$, conditional on the vector of states ω.

$$\mathbb{P}(\tilde{h}',\theta',x|\{s,a,h,\theta\}) = \mathbb{P}(j'|\omega)\times\mathbb{P}(\theta'|\{\omega\},j')\times\mathbb{P}(\tilde{h}',x|\{\omega\},j',\theta')$$

This tensor describes the discrete choice policy functions at iteration (i) of the algorithm.

A.3 Stationary distribution

After convergence on the decision rules, we compute the next period distribution using the deterministic and stochastic policy functions. Denote by λ_t the distribution across states at date t. The law of motion of the distribution is usually defined by:

$$\lambda_{t+1} = \mathbf{Q}'\times\lambda_t$$

Where $\mathbf{Q}$ is the transition matrix of dimensions (N_ω,N_ω), that is defined by the conditional probabilities of the next period states, given the current states and the choices. We decide to compute, for given the converged value function and prices, the transition matrix. This method is efficient as the conditional probabilities defining the discrete choices are of similar dimensions to the transition matrix, taken aside the exogenous shocks. We compute this operator following two steps.

Step 1: Endogenous movements We first compute the savings lotery procedure (Young, 2010), that consists into approximating off-grid liquid saving choices on grid, together with a probability to make that choice. We denote by $\tilde{a}'_{\text{cand}}$ the candidate savings policy function, that is defined as the array $\tilde{a}'_{\text{cand}}(s,\psi,h',\theta',x)$ and the associated lottery probabilities $\mathcal{P}_{\tilde{a}'|a'}$.

Therefore, we compute at each point of the state space, the probability to move on the endogenous states $(a', \tilde{h}', \tilde{\theta}')$, before any stochastic depreciation, using element-wise multiplication on previously computed matrices.

$$\mathbb{P}(\tilde{a}', \tilde{h}', \tilde{\theta}'|\omega) = \left[\sum_x \mathbb{P}(h', \theta', x|\omega)\right] \odot \mathcal{P}_{\tilde{a}'|a'}(\omega) \qquad \forall \tilde{a}'|a'$$

We store these elements in a temporary transition matrix, denoted $\mathbf{Q}^{(1)}$, for all income shocks in the grid.

Step 2: Exogenous movements We then turn into the "exogenous" movements. First, we apply the income transition matrix to the temporary transition matrix $\mathbf{Q}^{(1)}$. Then, we apply the stochastic depreciation to the temporary transition matrix. We obtain the following formula:

$$\mathbf{Q}(\omega, \omega') = \begin{cases} \mathbf{Q}^{(1)}(\omega, s', a', 1, 1) + \delta \times \left(\sum_{\tilde{\theta}'} \mathbf{Q}^{(1)}(\omega, s', a', 2, \theta)\right) & \text{For renters next period} \\ (1-\delta) \times \mathbf{Q}^{(1)}(\omega, s', a', h', \theta') + \delta \times \mathbf{Q}^{(1)}(\omega, s', a', h'+1, \theta') & \text{For owners next period} \end{cases}$$

Stationary distribution We solve for the stationary distribution as being the fixed point of the law of motion:

$$\lambda^* = \mathbf{Q}' \times \lambda^*$$

A.4 Market Clearing conditions with taste shocks

Definition of aggregates There are two types of aggregates: "begining of the period" and "mid-period". The aggregates at "mid" period are the ones that occur before depreciation, and on which markets clear. The beginning of the period aggregates are defined as the sum of the distribution across all states.

$$\text{Aggregate Housing Stocks} \qquad H_{\theta,t} = \sum_{h \in \mathcal{G}_h} h \times \left\{ \sum_s \int_a \lambda_t(s, a, h, \theta) \quad da \right\} \qquad \forall \theta$$

$$\text{Aggregate Asset holdings} \qquad A_t = \sum_{s,h,\theta} \int_a a \times \lambda_t(s, a, h, \theta) \quad da$$

Mid-period aggregates The mid-period aggregates are defined as the sum of the distribution across all states, before depreciation. For the housing stock, we denote by $H'_{\theta',t}$ the housing stock before depreciation at date t.

Mid-Period Housing Stocks:

$$H'_{\theta',t} = \sum_{k=1}^{N_h} h'_k \times \left\{ \sum_{s,h,\theta} \int_a \mathbb{P}^*_t(\{h' = h'_k\} \cap \theta' | \omega) \times \lambda_t(s,a,h,\theta) da \right\} \qquad \forall \theta'$$

Aggregate Rental Stocks

$$X_{\theta,t} = \sum_{k,l}^{N_h} (h'_k - x_l) \left\{ \sum_{s,h,\theta} \int_a \underbrace{\mathbb{P}^*_t \left(\{h' = h'_k\} \cap \{x = x_l\} \cap \{j_p = 3\} | \omega\right)}_{\text{Landlord}} \times \lambda_t(\omega) da \right\} \forall \theta'$$

Housing Market Clearing The housing market clears when the new supply of housing stock equals the net demand for housing stock (amount bought - amount sold). Denote by $\mathcal{M}^H_{\theta,t}$ the net demand for housing stock of type θ. The market clearing conditions at mid-period write

$$\mathcal{M}^H_{\theta,t} = 0 \qquad \Longleftrightarrow \qquad I_{\theta,t} = \text{Housing bought} - \text{Housing sold}$$

Therefore, the brown housing market clearing condition writes (same holds for the green housing market):

$$\begin{aligned} I_{\theta,t} = & \sum_{k=2}^{N_h} h'_k \cdot \left\{ \sum_s \int_a \mathbb{P}^*_t \Big(\{h' = h'_k\} \cap \{\theta' = b\} | \{s,a,0,b\} \Big) \times \lambda_t(s,a,0,b) da \right\} \\ & - \sum_{k=2}^{N_h} h_k \cdot \left\{ \sum_s \int_a \mathbb{P}^*_t \Big(\{h' = 0\} \cap \{\theta' = b\} | \{s,a,h_k,b\} \Big) \times \lambda_t(s,a,h_k,b) da \right\} \end{aligned}$$

The market clearing conditions write:

$$0 = I_{b,t} - \sum_{k=2}^{N_h} h_k \times \left\{ \sum_s \int_a \underbrace{\mathbb{P}^*_t \Big(\{h' = h_k\} \cap \{\theta' = b\} \Big)}_{\text{Buy brown}} \cdot \lambda_t(s,a,0,b) - \underbrace{\mathbb{P}^*_t \Big(\{h' = 0\} \cap \{\theta' = b\} \Big)}_{\text{Sell brown}} \lambda_t(s,a,h_k,b) d$$

$$0 = I_{g,t} - \sum_{k=2}^{N_h} h_k \times \left\{ \sum_s \int_a \underbrace{\mathbb{P}^*_t \Big(\{h' = h_k\} \cap \{\theta' = g\} \Big)}_{\text{Buy green}} \cdot \lambda_t(s,a,0,b) - \underbrace{\mathbb{P}^*_t \Big(\{h' = 0\} \cap \{\theta' = b\} \Big)}_{\text{Sell green}} \lambda_t(s,a,h_k,g) d$$

Rental Market Clearing The dwelling market clears when the supply of dwellings from landlords equals the demand for dwellings from renters.

$$0 = \sum_{k=2}^{N_h} \sum_{x_l \le h_k} (h_k - x_l) \times \left\{ \sum_{i \ge 2}^{N_h} \sum_{\theta'} \sum_{s} \int_a \mathbb{P}_t^* \Big(\{h' = h_k\} \cap \{\theta'\} \cap \{x = x_l\} \Big) \lambda_t(s, a, h_i, b) da \right\}$$
$$- \sum_l x_l \times \left\{ \sum_{h,\theta} \sum_s \int_a \mathbb{P}_t^* \Big(\{h' = 0\} \cap \{\theta' = b\} \cap \{x = x_l\} \Big) \lambda_t(s, a, h, \theta) da \right\}$$

When the rental market clears, the total mid-period housing stock equals the dwelling usage in the economy:

$$0 = \sum_{s,h,\theta} \int_a \lambda_t(s, a, h, \theta) \times \sum_{\theta'} \left[\sum_{k=2}^{N_h} h_k \cdot \mathbb{P}_t^* \big(\{h' = h_k\} \big) - \sum_l^{N_x} x_l \cdot \mathbb{P}_t^* \big(\{x = x_l\} \big) \right] da$$

Algorithm We clear the three markets of the economy (together with the government market clearing) using a quasi-Newton algorithm.